Reasoning about Time and Reliability

Probabilistic CTL model checking

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Seminar „Theorie und Anwendung von Model Checking“

Outline

Motivation

- Communication networks need to be error free and reliable. In addition, they often operate under real time conditions, meaning they must meet certain deadlines in order to work correctly.
- Particularly, this becomes vital when wireless networks are used – where significant packet loss is inevitable.
- In this talk, a temporal logic will be presented in which propositions such as "There is a probability of at least 99% that a message is received at most 6*ms* after it is sent." can be expressed.

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An extension to CTL with time and probability

- Probabilistic real time CTL (PCTL) is an extension to the well-known temporal logic CTL (Computation Tree Logic).
- PCTL extends this concept by both a discrete time structure allowing real time statements as well as probabilities for these events by which hard and soft deadlines can be modeled.
- There exist polynomial time model checking algorithms suitable for small structures.

Outline

Syntax

Definition

- Atomic propositions are state formulae.
- If φ and ψ are state formulae, then so are $\neg\varphi$, $(\varphi \wedge \psi)$, $(\varphi \vee \psi)$ and $(\varphi \rightarrow \psi)$.
- If φ and ψ are state formulae and $\tau \in \mathbb{N} \cup \infty$, then $(\varphi \ \mathrm{U}^{\leq\tau} \ \psi)$ and $(\varphi \ \mathrm{W}^{\leq\tau} \ \psi)$ are path formulae.
- If F is a path formula and $\rho \in [0, 1]$, then $[F]_{\geq\rho}$ and $[F]_{>\rho}$ are state formulae.

We will use $\varphi \ \mathrm{U}^{\leq\tau}_{\geq\rho} \ \psi$ as shorthand for $[\varphi \ \mathrm{U}^{\leq\tau} \ \varphi]_{\geq\rho}$.

PCTL: some intuition

- $\varphi \; \mathrm{U}^{\leq 4}_{\geq 0.99} \; \psi$ means there is a probability of at least 99% that both ψ will come true within 4 time units and φ holds till ψ comes true.
- $\varphi \; \mathrm{W}^{\leq 4}_{\geq 0.99} \; \psi$ means there is a probability of at least 99% that either the above condition holds or φ holds for at least 4 time units.
- If the underlying structure is modelled wisely, "time units" can be substituted with real time units such as "seconds" or "milliseconds".

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Expressing CTL through PCTL

Example

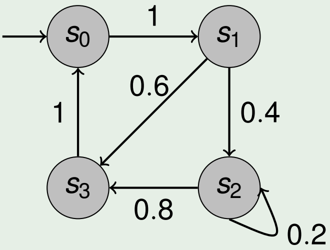
Since PCTL extends CTL, its properties can be expressed by extreme values of time and probability.

- generally: $\mathrm{G}\varphi \equiv \varphi \,\mathrm{W}^{\leq\infty}$ *false*
- finally: $\mathrm{F}\varphi \equiv$ *true* $\mathrm{U}^{\leq\infty}\, \varphi$
- until operator: $\varphi \mathrm{U} \psi \equiv \varphi \,\mathrm{U}^{\leq\infty}\, \psi$
- universal path quantifier: $\forall F \equiv [F]_{\geq 1}$
- existential path quantifier: $\exists F \equiv [F]_{>0}$

Logic structure

The structure over which PCTL formulae are evaluated is a finite automaton with labels on states and probabilistic transitions. Each transition corresponds to one time step.

Example



Formal definition of the structure

Definition

A structure $\mathcal{K}$ is a tuple $(\mathcal{S}, s_\perp, \mathcal{T}, L)$ where

- $\mathcal{S}$ is a finite set of states,
- $s_\perp \in \mathcal{S}$ is the initial state,
- $\mathcal{T}$ is a probabilistic transition function $\mathcal{T} : \mathcal{S}^2 \to [0, 1]$, such that $\sum_{t \in \mathcal{S}} \mathcal{T}(s, t) = 1$ for all states $s \in \mathcal{S}$ and
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Prerequisites for semantics

Definition

1. A path beginning in state s_0 is an infinite sequence $(s_0, s_1, \ldots)$. Let the set of paths (beginning in s_0) be $\mathcal{P}(s_0)$.
2. The n-th state of a path π is denoted $\pi[n]$, a finite prefix of length n is denoted $\pi|_n := (s_0, \ldots, \pi[n])$.
3. For each state s a probability measure $\mu_s : \mathcal{P} \to [0,1]$ is defined for each finite sequence $(s_0, \ldots, s_n)$ by

$$\mu_s(\{\pi \in \mathcal{P} : \pi|_n = (s_0, \ldots, s_n)\}) = \prod_{i=1}^{n} \mathcal{T}(s_{i-1}, s_i)$$

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Semantics

Definition

$s \models a$ iff $a \in L(s)$

$s \models \neg\varphi$ iff $s \not\models \varphi$

$s \models \varphi \wedge \psi$ iff $s \models \varphi$ and $s \models \psi$

$\pi \models \varphi\ \mathrm{U}^{\leq\tau}\ \psi$ iff there is a $\iota \leq \tau$ with $\pi[\iota] \models \psi$ and $\pi[\kappa] \models \varphi$ for all $\kappa < \iota$

$\pi \models \varphi\ \mathrm{W}^{\leq\tau}\ \psi$ iff $\pi \models \varphi\ \mathrm{U}^{\leq\tau}\ \psi$ or $\pi[\kappa] \models \varphi$ for all $\kappa \leq \tau$

$s \models [F]_{\geq\rho}$ iff $\mu_s(\{\pi \in \mathcal{P}(s) \mid \pi \models F\}) \geq \rho$

Finally, we define $\models_{\mathcal{K}} \varphi$ iff $s_\perp \models \varphi$.

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Outline

1. Probabilistic real time CTL
2. PCTL model checking
3. Calculation example

A model checking algorithm for PCTL

- In this section a model checking algorithm will be presented, which determines whether a given structure $\mathcal{K}$ models some formula φ, i.e. $\models_{\mathcal{K}} \varphi$.
- When it finishes, each state will be labeled with the complete set of subformulae of φ which hold in that state. If the initial state $s_{\perp}$ is labeled with φ, then the structure is a model for φ.
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Labeling states with formulae

1. We define an extended labeling function $\mathcal{L}$ for every subformula of φ. Initially, all states are labeled with atomic propositions: $\forall s \in \mathcal{S} : \mathcal{L}(s) := L(s)$
2. Suppose the subformulae of φ have been ordered in size (of logic connectives). Then from the smallest to φ itself change the labels of all states:
   - For a subformula $\neg\psi$ set $\mathcal{L}(s) := \mathcal{L}(s) \cup \{\neg\psi\}$ if $\psi \notin \mathcal{L}(s)$
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Labeling for $(\varphi_1 \, \mathrm{U}^{\leq \tau}_{\geq \rho} \, \varphi_2)$

Assume $\tau < \infty$. Let $s_1, \ldots, s_n$ be the states of $\mathcal{S}$.

- Define a $n \times n$ transition matrix $A = (\alpha_{ij})$ by

$$\alpha_{ij} = \begin{cases} \mathcal{T}(s_i, s_j) & \text{if } \varphi_1 \in \mathcal{L}(s_i) \text{ and } \varphi_2 \notin \mathcal{L}(s_i) \\ 1 & \text{else if } i = j \\ 0 & \text{otherwise} \end{cases}$$

  By this, some transition probabilities are given: Rows represent the “from-states” and columns represent the “to-states”.
- Thus, the matrix A^τ intuitively gives transition probabilities for τ transition steps.

Labeling for ($\varphi_1 \; \mathrm{U}^{\leq \tau}_{\geq \rho} \; \varphi_2$) (contd.)

- Define a vector $P = (p_k)$ of size n with

$$p_k = \begin{cases} 1 & \text{if } \varphi_2 \in \mathcal{L}(s_k) \\ 0 & \text{otherwise} \end{cases}$$

  Intuitively, this represents the set of states in which φ_2 (the postcondition) is true.
- Calculate the vector $P' := A^{\tau} \cdot P$. This represents the probabilities of each state modeling the until-formula.
- Thus, for each state s_k redefine $\mathcal{L}(s_k) := \mathcal{L}(s_k) \cup \{\varphi_1 \; \mathrm{U}^{\leq \tau}_{\geq \rho} \; \varphi_2\}$ if the k-th entry in P' is at least ρ.

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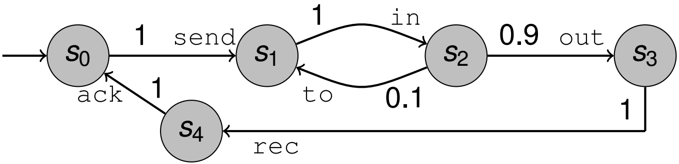
Labeling in special cases

- The case with $\tau = \infty$ must be treated separately.
- Other extreme cases can be calculated with separate algorithms for optimization reasons.
- The weak until operator W can be transformed into a formula with just U.
- There is another algorithm for calculation of P' with slightly different complexity.

Outline

Verification of a communication protocol

As an example, we will now verify a simple (fictional) communication protocol. It provides error free communication from a sender to a receiver over a lossy medium. We assume that acknowledgements are never lost though.



We also assume the message loss probability to be 10%.

Properties of interest

- An interesting question to pose is, if the system is able to meet certain deadlines, for example if there is a probability of at least 99% that a message is received (s_4) at most 6 time units after it is send (s_0).
- For a single event, this can be expressed by

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Proving the assertion

- First of all, we translate the formula into a "pure until-formula" and name the subformulae:

$$\varphi = \underbrace{s_0}_{\varphi_1} \rightarrow \underbrace{\left(\underbrace{true}_{\varphi_2}\ \mathrm{U}^{\leq 6}_{\geq 0.99}\ \underbrace{s_4}_{\varphi_3}\right)}_{\varphi_4}$$

- Now states will labeled with atomic propositions:
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Construction of matrix A and vector P

We construct the matrix A and the vector P according to the definition given earlier:

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0.1 & 0 & 0.9 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \qquad P = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

Labeling for $\varphi_4 = \varphi_2 \, \mathrm{U}^{\leq 6}_{\geq 0.99} \, \varphi_3$

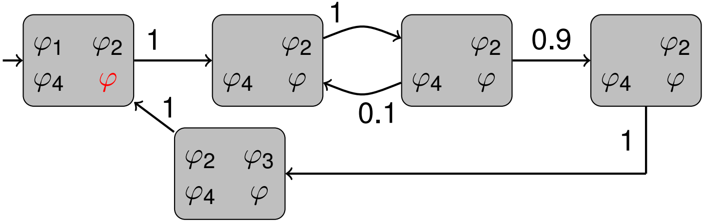
Next, we calculate $P' = A^6 \cdot P$:

$$A^6 = \begin{pmatrix} 0 & 0 & 0.01 & 0 & 0.99 \\ 0 & 0.001 & 0 & 0.009 & 0.99 \\ 0 & 0 & 0.01 & 0 & 0.99 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \qquad P' = \begin{pmatrix} 0.99 \\ 0.99 \\ 0.99 \\ 1 \\ 1 \end{pmatrix}$$

Since every entry in P' is at least 0.99, we conclude that every state is to be labeled with φ_4.

The initial state is labeled with φ

Finally, every state is labeled with $\varphi = (\varphi_1 \rightarrow \varphi_4)$ since every state is labeled with φ_4.



Since the initial state s_0 is labeled with φ, we have shown that the given protocol is a model for our assumption.

Summary

- The motivation for a logic like PCTL stems largely from an application view.
- We have seen practical usefulness for PCTL-expressible properties.
- The model checking algorithm shown is implemented in the PRISM model checking tool.

H. Hansson and B. Jonsson.
A logic for reasoning about time and reliability.
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