

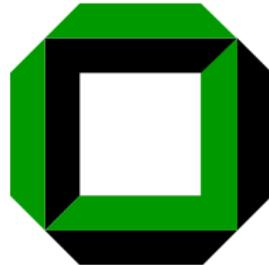
Formale Entwicklung objektorientierter Software

Praktikum im Wintersemester 2007/2008

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12. Dezember 2007



Loop Invariants



Unwinding Loops

$$\text{loopUnwind} \longrightarrow \frac{}{\Gamma \implies \mathcal{U}\langle \pi \text{ while}(e) \ p \ \omega \rangle \phi, \ \Delta}$$



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How to handle a loop with...

- 0 iterations?



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How to handle a loop with...

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- 10 iterations?



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- 10 iterations? Unwind 11×.



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- 10000 iterations? Unwind 10001× (and don't make any plans for the rest of the day).



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- 0 iterations? Unwind 1×.
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 - 10000 iterations? Unwind 10001× (and don't make any plans for the rest of the day).
 - an **unknown** number of iterations?
- ~~ We need an **invariant rule**. (or some other form of induction)



Basic Invariant Rule

$$\text{loopInvariant} \quad \frac{}{\Gamma \implies \mathcal{U}[\pi \text{ while}(e) \ p; \omega]\phi, \Delta}$$



Basic Invariant Rule

$$\Gamma \implies \mathcal{U}Inv, \Delta \quad (\text{initially valid})$$

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Basic Invariant Rule

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- Context $\Gamma, \Delta, \mathcal{U}$ must be omitted in 2nd and 3rd premiss



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- Context contains (parts of) precondition and class invariants
- Required context information must be added to loop invariant Inv



Example

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int i = 0;  
while(i < a.length) {  
    a[i] = 0;  
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Precondition: $a \neq \text{null}$

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- But: determining unaffected formulas syntactically is impossible because of aliasing
- Solution: **anonymising updates** \mathcal{V} delete information about modified locations

```
 $\mathcal{V} = \{i := i_0 \mid \text{for } x; a[x] := arr_0(a, x)\}$ 
```



Improved Invariant Rule

loopInvariant ————— $\Gamma \implies \mathcal{U}[\pi \text{ while}(e) \ p; \omega]\phi, \Delta$



Improved Invariant Rule

$$\Gamma \implies \mathcal{U}Inv, \Delta$$

(initially valid)

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 - $\mathcal{V} = \{ * := * \}$ wipes out **all** information



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- For assignable \everything (the default):
 - $\mathcal{V} = \{* := *\}$ wipes out **all** information
 - Equivalent to basic invariant rule
 - **Avoid this!** Always give a more narrow assignable clause.



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$$\{i := i_0\}anon \rightarrow [i=5;]\{i := i_0\}anon$$



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↷ Add $\mathcal{V}anon$ to *Inv* in “preserved” case.



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$$v = a.length - i$$



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↷ - Add $v \geq 0$ to Inv



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- Add $v < v^{old}$ to right of “preserved” case



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- Add $v < v^{old}$ to right of “preserved” case
- Replace box with diamond



Further Complications

Since Java is a **real** language...

- the loop guard expression e may have side effects



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- the loop body may use break or continue

↝ The invariant rule in KeY takes all this into account.



Loop Specifications in JML

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int i = 0;  
/*@  
 @  
 @  
 @  
 @  
 @*/  
while(i < a.length) {  
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@ decreases a.length-i;  
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Using Method Contracts



Expanding Method Bodies

$$\text{methodBodyExpand} \quad \frac{}{\Gamma \implies \mathcal{U}\langle \pi \circ.m(p_1, \dots, p_n)@C; \omega \rangle\phi, \Delta}$$



Expanding Method Bodies

$$\text{methodBodyExpand} \frac{\Gamma \implies \mathcal{U}\langle\pi \text{ method-frame}(C,o)\{b\} \omega\rangle\phi, \Delta}{\Gamma \implies \mathcal{U}\langle\pi o.m(p_1, \dots, p_n)@C; \omega\rangle\phi, \Delta}$$



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Disadvantages:

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Disadvantages:

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- Duplication of effort
- Can lead to huge proofs



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Disadvantages:

- Non-modular
- Duplication of effort
- Can lead to huge proofs
- Sometimes the method body is not available (e.g. native methods)



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Given a contract $(Pre, Post, \mathcal{V})$:



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Given a contract $(Pre, Post, \mathcal{V})$:

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(Pre)

$$\Gamma \implies \mathcal{U}\mathcal{V}(\text{exc} \doteq \text{null} \wedge Post \rightarrow \langle\pi \omega\rangle\phi), \Delta$$

(Post)

$$\Gamma \implies \mathcal{U}\langle\pi \circ.m(p_1, \dots, p_n)@\mathcal{C}; \omega\rangle\phi, \Delta$$



Use Method Contract Rule

Given a contract $(Pre, Post, \mathcal{V})$:

$$\frac{\Gamma \implies \mathcal{U}Pre, \Delta \quad (\text{Pre}) \\ \Gamma \implies \mathcal{U}\mathcal{V}(\text{exc} \doteq \text{null} \wedge Post \rightarrow \langle\pi \omega\rangle\phi), \Delta \quad (\text{Post}) \\ \Gamma \implies \mathcal{U}\mathcal{V}(\text{exc} \neq \text{null} \wedge Post \rightarrow \langle\pi \text{ throw exc; } \omega\rangle\phi), \Delta \quad (\text{Exc.})}{\Gamma \implies \mathcal{U}\langle\pi \circ.m(p_1, \dots, p_n)@\mathcal{C}; \omega\rangle\phi, \Delta}$$



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END**

