Tableaux + Constraints

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Abstract. There is an increasing number of publications in which the analytic tableaux calculus is combined with technology based on constraint solving. Although the details, as well as the purpose of these combinations vary widely, the results are invariably referred to as "constraint tableaux" or sometimes "constrained tableaux". We review some of the combinations and propose a more differentiated nomenclature.

1 Classification of Approaches

In this paper, we present an overview of previous work which in some way uses constraints in an analytic tableau framework. We shall list all the approaches we are currently aware of. If anybody knows of relevant publications not mentioned here, the authors would be happy to include them.

We classify the various approaches according to the location *where* constraints are used in the tableau, in a quite syntactical way. Semantically of course, this corresponds to *what* is being constrained in each particular case, so it is not too surprising that approaches in the same categories seem to use constraints for related purposes. Note that we use the word "constrained" as past participle of the verb "to constrain", while a "constraint" is a noun, denoting an entity used to constrain a thing.

We understand our nomenclature as a first proposal and a basis for further discussion.

2 Constrained Tableaux

The most obvious entity that might be constrained is, of course, the whole tableau. We propose to call such combinations simply *constrained tableau* calculi.

Constrained tableaux are used by Degtyarev and Voronkov [1, 2] in their calculi for ordered superposition based equality handling. The constraints are mainly used to record ordering restrictions between the ground instantiations of free variables. But one also sees another interesting use of constraints, namely that unification constraints for branch closure are added to the global constraint instead of applying substitutions globally. For instance, given two literals $p(X)$, $\neg p(a)$ on the same branch, a constraint $X \equiv a$ would be generated

separately from the formulae on the tableau. In a system using unification, the substitution $[X/a]$ would be applied globally. This might enable rewriting steps on other branches, which can shown to be redundant.

Another approach that uses a global constraint on free variable instantiations is employed in the theorem prover SETHEO [12]. Here, the idea is to check regularity of tableaux by gathering constraints which ensure that the free variables are not instantiated in a way that renders two formulae on the same branch identical. Accordingly, this approach uses 'dis-unification' constraints.

3 Constrained Formula Tableaux

Instead of accumulating constraints globally, it is possible to add them to the formulae or signed formulae on a tableau branch. We will then speak of *constrained formula* tableaux.

For instance, if a rule application calls for some substitution $[X/a]$, a constrained tableau method would typically add the constraint $X \equiv a$ to the global constraint. In a constrained formula tableau, the constraint $X \equiv a$ is instead added to every new formula produced by the rule application. One also usually has *constraint propagation*, which means that the constraints of the formulae in the premisses of a rule application accumulated in the constraint of the conclusion.

This approach was used in several papers by Giese for equality handling [7, 6] and simplification rules [4, 8, 3] in a non-backtracking context. The property of constrained formula tableaux exploited in this context is that a rule which merely adds a constrained formula to a tableau branch never introduces a backtracking point, while a rule which modifies a global constraint usually does.

4 Constrained Branch Tableaux

In between constrained tableau and constrained formula tableaux one has the possibility to attach constraints to the branches of a tableau. We call this approach *constrained branch* tableaux.

This was done by van Eijck [19] for constraints on free variable instantiations with a similar intention as described above for constrained formulae. A branch containing a constraint C that is not satisfied by the instantiation for the free variables may be regarded as closed. As formulae within branches are conjunctively connected, and most constraint languages are closed under intersection, this does usually not result in greater expressivity, but gives more succinct calculi, when branch closure rules are more complicated than a simple test for a complementary pair. Note that these constraints on branches could be simulated by attaching their negations to "false" literals in a constrained formula approach.

To illustrate the constrained branch approach, assume again that a certain tableau expansion requires a substitution $[X/a]$. Van Eijck handles this with constrained branch tableaux by splitting the branch on which the rule would

be applied. On the first branch, the rule is actually applied, performing the necessary substitution X/a on the result. On the second branch, the constraint $X \not\equiv a$ is added. The effect is that if the final closing substitution substitutes a for X, the constraint on the second branch becomes false and closes it, so the proof on the (probably simpler) first branch is sufficient. Otherwise, a proof has to be found on the second branch, which does not profit from the rule application.

Note that there are probably efficiency problems with this approach. Take n possible, unrelated rule applications, each requiring a different instantiation. While these can be applied consecutively in any order in a constrained formula tableau, the constrained branch approach generates 2^n branches to account for all combinations of constraint satisfaction.

A very different kind of constrained branch tableau was proposed by Hähnle and Ibens [10], and later refined by Goubault and Schmitt [9]. These contributions deal with labeled tableau systems for linear temporal logic. Integer ordering constraints are used to keep track of the required ordering of points in time. Branches can be closed exactly if they contain unsatisfiable constraints. In other words the question of branch closure is delegated to the constraint system. This reinforces the point made above that constrained branch tableaux are best used in situations, where branch closure is very complicated.

5 Constrained Subformula Tableaux

To our knowledge, the smallest syntactic entities that have had constraints attached to them are subformulae of the formulae in tableaux. Such *constrained subformula tableaux* have been proposed by Peltier [15, 16]. Instead of just juxtaposing formulae and constraints, Peltier intertwines them. For instance, in a formula

$$\forall x.\forall y.(x = y \vee p(x, y)) \quad ,$$

$x = y$ plays the role of a constraint, which makes the formula $p(x, y)$ available only if x and y are instantiated by different ground terms. The symbol $=$ denotes *syntactic* equality, and not an equality predicate in the usual sense. On the other hand, quantification over the variables x, y used in the syntactic equality constraint is possible. This means that the semantics of such a mixture of formulae and constraints can only be defined with respect to models, in which the elements of the carrier set are ground terms (such as Herbrand models). The possibility of attaching different constraints to different parts of a larger formula is a potential advantage of Peltier's approach.

6 Constraint Merging Tableaux

There is also a family of approaches which don't necessarily add constraints to the tableaux at all. The constraints are used as an intermediate representation to find out whether a tableau is closed. This invariably requires some kind of merging operation on sets of closing instantiations for different sub-tableaux. As

these sets are of course represented by some form of constraints, one might as well talk about *constraint merging tableaux*.

Constraint merging tableaux have been introduced in the incremental closure technique of Giese [5, 8]. Van Eijck attempted an implementation in a lazy functional programming language [17, 18], but the resulting provers were unfortunately incomplete. A successful functional implementation was recently given by Sörensson and Hähnle [11].

Constraint merging tableaux can be combined very nicely with constrained formulae. First, constraint merging tableaux are used to avoiding backtracking, and constrained formula calculi don't require backtracking, as we noted earlier. And second, as tableau closure is formulated and checked using constraints anyway, having constraints on the formulae hardly means any extra complexity.

7 Constraint Tableaux

There is finally one approach[14] in which the whole tableau is actually replaced by a potentially infinite but lazily computed constraint. Here, the constraint *is* the tableau, so we can finally talk about a *constraint tableau*.

There are certain difficulties in finding a correct definition for constraint semantics and satisfiability testing for lazily computed infinite constraints. On the other hand, once correct definitions are found, it might turn out that one can reason algebraically on constraints in a way that is not obvious from a tableau representation.

Note that the satisfiability check as presented by Ó Nualláin corresponds quite closely to the ones described under "Constraint Merging Tableaux". Still, this might not necessarily be implied by the constraint tableau idea. That's why we propose keeping them in a separate section.

8 As Yet Unclassified Approaches

We have come across one more line of work where the term "constraint propagation" is used in a tableau context, namely that of Massacci [13]. What Massacci means by that term seems to be essentially unit propagation, using simplification rules. There are certainly no separate syntactical entities referred to as constraints in this work, so we chose to keep it separate from the other techniques presented in this paper.

9 Conclusion

We attempted to give an overview of the existing literature on combinations of tableaux and constraint systems. We proposed a systematic, differentiated nomenclature, which might be used in the future to distinguish between the different ideas and methods.

We hope to receive comments from the community, both on work we failed to mentioned here, and on the proposed nomenclature.

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